

DISTRIBUTED CONTROL TOPOLOGIES FOR DEEP SPACE FORMATION FLYING SPACECRAFT

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ABSTRACT – *A formation of satellites flying in deep space can be specified in terms of the relative satellite positions and absolute satellite orientations. The redundancy in the relative position specification generates a family of control topologies with equivalent stability and reference tracking performance, one of which can be implemented without requiring communication between the spacecraft. A relative position design formulation is inherently unobservable, and a methodology for circumventing this problem is presented. Additional redundancy in the control actuation space can be exploited for feed-forward control of the formation centroid's location in space, or for minimization of total fuel consumption.*

1 - INTRODUCTION

The collective behavior of spacecraft flying in formation can be used to synthesize instruments of greater utility than could otherwise be achieved with a single spacecraft. One example, which motivates much of our work, is an interferometric imaging system composed of multiple spacecraft. Several interferometric flight projects, based on formation flying, have been proposed and studied including Darwin [1], Terrestrial Planet Finder (TPF) [2] and StarLight [3].

We use the interferometric imaging application as a basis for discussing formation control problems which are applicable to a wider range of problems. Figure 1a illustrates a conceptual interferometric imaging configuration. Each spacecraft acts as a collector, reflecting light from the imaging source to a combiner spacecraft. The light from any two collectors is combined at a detector and, if the optical pathlengths are held fixed, an interference pattern is generated. Each measurement of the amplitude and phase of the mutual coherence between the two reflected light beams amounts to a sample of the spatial Fourier transform of the image. Multiple measurements gives sufficient data to allow reconstruction of the image. The advantage of imaging in this way is that the effective aperture depends on the collector separation. Future objectives call for separations of the order of kilometers, giving resolutions that cannot be matched by any monolithic spaceborne telescope.

Our work focuses on deep space missions, in which the formation is in heliocentric orbit rather than earth orbit. In this scenario formation control problems (e.g. initialization, reorientation, resizing, tracking, station keeping, etc.) can be specified in terms of the tracking of relative spacecraft position and spacecraft attitude. We make some assumptions specifically tailored to deep space applications. The most significant of these is that the spacecraft can sense their relative position but not their absolute position.

The spacecraft in the formation are free flying and their dynamics are coupled only through the implementation of control to meet the application objectives. To maintain the performance of the formation it is necessary to maintain the relative position and absolute orientation of the spacecraft. Actuation for control purposes is performed on the individual spacecraft.

There are many possible topologies for sensing, control, and communication within a formation. Communication bandwidths, synchronization constraints, and sensor capabilities affect

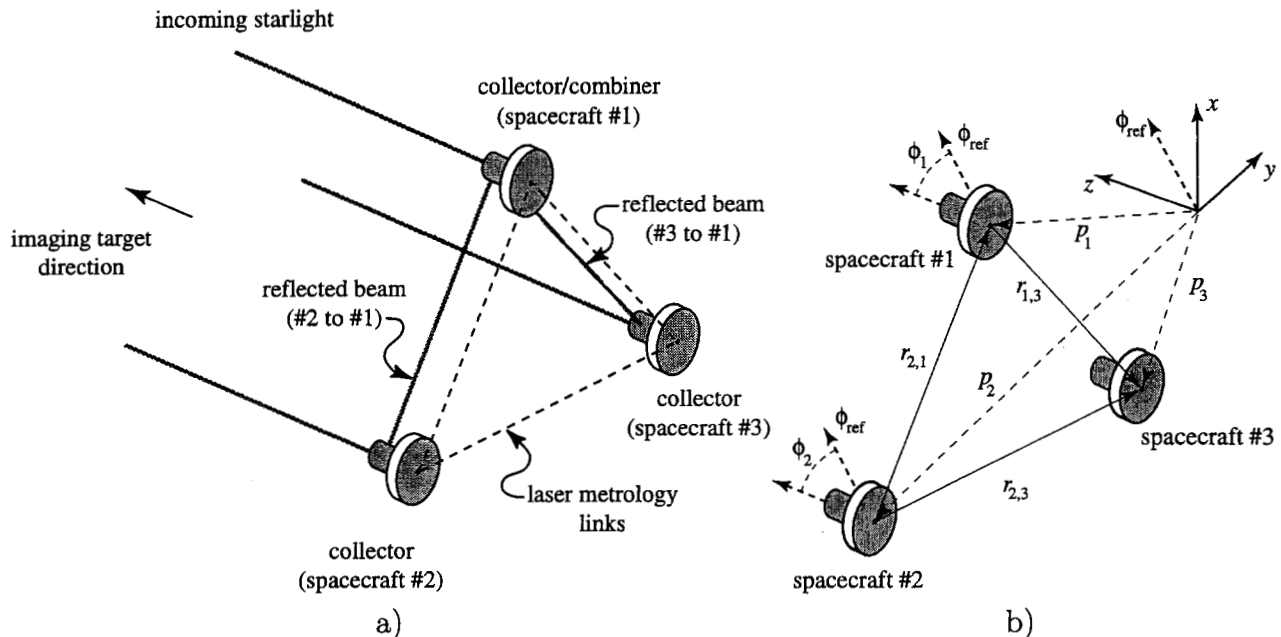


Fig. 1: a) Interferometric imaging configuration using multiple spacecraft in formation. b) Definition of the inertial and relative position variables.

the performance of any chosen topology. These issues have been studied for various topologies [4, 5, 6, 7, 8, 9].

The work presented here emphasizes techniques that allow flexibility in the specification of the formation topology. This will ultimately allow us to develop formation systems that are robust to missing measurements, lost communication links, and, in some cases, faulty spacecraft. Being able to switch between communicated and measured information also allows us to investigate the tradeoffs involved in implementing decentralized topologies. By decentralized topologies we mean those in which the communicated and/or measured information available to the spacecraft is only a subset of the full set of formation variables. See [10] and the references therein for a discussion of decentralized control in spacecraft formations.

Our prior work [11] developed a characterization of the redundancy in a relative position based architecture and used this to construct algorithms for independently switching between measurements and communicated information on each spacecraft. The stability and equivalent performance of this approach was proven, and it was shown that there exists a topology achieving a global tracking objective which requires no communication between the spacecraft.

We briefly review this prior work, and then proceed to develop a methodology for designing formation controllers based on relative position measurements. Under certain conditions it is possible to exploit a degree of freedom in the actuation in order to meet other formation objectives. We provide a formulation for two such objectives: minimizing total fuel consumption; and maintaining the position and velocity of the formation centroid in inertial space. The formulation allows a wide range of design methods to be applied. We present one suitable method based on Linear Matrix Inequality (LMI) optimization.

2 - FORMATION DEFINITION AND SENSING

We begin by considering a typical formation and defining the notation associated with the various inertial and relative position and attitude variables. Consider a formation of N spacecraft, and for simplicity it is sufficient here to define on each a reference attitude, ϕ_i , $i = 1, \dots, N$, (defined in any complete attitude parametrization) with respect to an inertially fixed direction, ϕ_{ref} . Each spacecraft is located at position $p_i = [x_i, y_i, z_i]^T$ (the T superscript denotes transpose) within an inertial frame. The relative position between each two spacecraft is defined

as,

$$r_{i,j} = p_j - p_i = \begin{bmatrix} x_j & y_j & z_j \end{bmatrix}^T - \begin{bmatrix} x_i & y_i & z_i \end{bmatrix}^T, \quad i, j = 1, \dots, N, \quad i \neq j.$$

Naturally, $r_{i,j} = -r_{j,i}$, and in an N spacecraft formation there are $N(N-1)/2$ relative three dimensional distances that can be defined modulo the opposite direction equivalences. Figure 1b illustrates these definitions.

In deep space, an accurate measurement of (x_i, y_i, z_i) is not available. On the other hand, the $r_{i,j}$ variables can be precisely measured by a combination of radio frequency and laser metrology based sensors. In contrast to absolute position, spacecraft attitude can be measured to very high accuracy using standard on-board sensors.

The formation is defined by the attitude of each spacecraft, $\{\phi_i, i = 1, \dots, N\}$, and their relative positions $\{r_{i,j}, i, j = 1, \dots, N, i \neq j\}$. Accurate measurements of these variables are available; an accurate measurement of the absolute location of the formation is not. There is some redundancy in the $r_{i,j}$ measurements as the $N(N-1)/2$ relative positions are not independent. We exploit this redundancy in developing control topologies that do not require all relative positions to be measured.

3 - FORMATION CONTROL

3.1 - Attitude Control

A formation geometry specification includes the attitude of each spacecraft, ϕ_i , with respect to an inertial frame. In deep space mission applications, each spacecraft is assumed to have a local measurement of its attitude which gives the option of implementing strictly local attitude controllers. However, this may not be optimal with respect to the formation objectives during synchronized maneuvers and this issue will be investigated in future research. The focus of the current paper is the control of the relative positions, $r_{i,j}$.

3.2 - Relative Position Control

The full set of relative position measurements contain redundancies that can be expressed as algebraic constraints. For example, $r_{i,j} + r_{j,k} + r_{k,i} = 0$. For the formation to be well defined these constraints must also apply to the relative position commands. Note that analogous constraints also apply to the errors and disturbances.

We express this constraint in the form,

$$\begin{bmatrix} r_{1,2} \\ r_{1,3} \\ \vdots \\ r_{N-1,N} \end{bmatrix} = C \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix} = \begin{bmatrix} -I & I & 0 & \dots & 0 \\ -I & 0 & I & \dots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ 0 & \dots & & -I & I \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix}.$$

and note that the matrix C is not full row rank. Therefore, there exists a matrix, M , satisfying, $\begin{bmatrix} r_{1,2}^T & \dots & r_{N-1,N}^T \end{bmatrix} M = 0$. This is a convenient method of expressing the algebraic redundancies in the relative position measurements.

4 - EQUIVALENT TOPOLOGIES

We will use the redundancy characterized above to define a class of linear transformations that have the effect of removing specified relative measurements from the controller. These transformations are of the form $H = I - XM^T$, where X is any matrix satisfying $M^T X = I$. This has the effect of expressing some of the relative position measurements as linear combinations of the others. If we are given a controller K , designed to use all of the relative measurements, r , then the transformed controller, $\hat{K} = KH$, uses only a subset of the relative position measurements. The particular subset depends on our choice of X . It is not true that $KH = K$. Our prior work [11] showed that KH has the same formation stability and tracking performance properties as the global controller K .

This approach can be further generalized. The i^{th} row of the operator K is the controller that spacecraft i must implement. We can define a different transformation, H_i , for each spacecraft and still maintain formation stability and performance. To formalize this, partition the identity matrix into q block diagonal pieces via, $I = \sum_{i=1}^q E_i E_i^T$, where the vector E_i has only ones in the components which define the i^{th} partition. Now define the transformed controller via, $\hat{K} = \sum_{i=1}^q E_i E_i^T K H_i$, where the H_i are transformations of the form, $H_i = I - X_i M_i^T$. This has the effect of grouping the controller outputs into q disjoint groups, and applying a different input transformation, H_i , to each. The formation stability using this \hat{K} is proven in [11].

4.1 - Local Relative Control Topology

The analysis tools summarized above can be used to prove the existence of a particular topology for formation flying. We define this topology as follows.

Definition 1 *A control topology in which all actuation signals depend only on relative measurements with respect to the actuation location is termed a local relative control topology.*

In our application this topology means that all control calculations can be performed locally, based only on local relative measurements. In other words, the calculation of the actuation for the i^{th} spacecraft, u_i , depends only on $r_{i,j}$, $j = 1, \dots, N$, $j \neq i$. This topology is interesting in that it can be implemented without any communication between the spacecraft. See [11] for a constructive proof that such a topology can always be constructed from any stable formation controller satisfying a global formation design objective.

5 - FORMATION CONTROL DESIGN

The above motivates us to consider the design of global formation controllers based only on relative position measurements, and we now study this aspect in detail.

5.1 - Relative Position Based Formation Control

We consider a linear, state-space description of the spacecraft dynamics,

$$\dot{x} = A x + B u, \quad r = \begin{bmatrix} C & 0 \end{bmatrix} x.$$

Because the spacecraft are not physically coupled, A and B have a sparse block structure. The output matrix, C , gives the relative position measurements effectively coupling the spacecraft.

The first obstacle to design is that the state, x , is not fully observable from the relative measurements, r . Physically this arises from the fact that the position and velocity of the formation centroid cannot be determined by relative position measurements. To obviate this we use a similarity transformation of the state, $Tx = \begin{bmatrix} z & v \end{bmatrix}$, to give,

$$\begin{bmatrix} \dot{z} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} A_z & A_{zv} \\ 0 & A_v \end{bmatrix} \begin{bmatrix} z \\ v \end{bmatrix} + \begin{bmatrix} B_z \\ B_v \end{bmatrix} u, \quad r = \begin{bmatrix} 0 & C_v \end{bmatrix} \begin{bmatrix} z \\ v \end{bmatrix},$$

where (C_v, A_v) is observable. We note that the observable part of the dynamics,

$$\dot{v} = A_v v + B_v u, \quad r = C_v v,$$

can be used to design a formation controller using relative position measurements. Various control design methods can be applied at this point. We present one based on LMI optimization [12] for estimator and state-feedback design.

The state-feedback design problem is formulated in terms of finding a controller that drives all states within an initial ellipsoid, $\mathcal{V}_0 = \{v \mid v^T V_0 v < 1, V_0 = V_0^T > 0\}$, to zero with a bounded cost given by,

$$\|W_v v\|_2^2 + \|W_u u\|_2^2 \leq \gamma^2.$$

Note that we have chosen to independently penalize both the state error and the control action via the symmetric positive definite weighting matrices W_v and W_u respectively. Finding the minimum γ clearly gives the optimal controller for solving this problem. The controller is given by the following LMI optimization problem.

$$\begin{aligned} \min_{\gamma, Q, Y} \quad & \gamma \quad \text{subject to: } \gamma > 0, \quad Q = Q^T > 0, \\ & \begin{bmatrix} \gamma^2 I & V_0^{-1/2} \\ V_0^{-1/2} & Q \end{bmatrix} > 0, \\ \text{and} \quad & \begin{bmatrix} -(QA_v^T + A_v Q + Y^T B_v^T + B_v Y) & Q & Y \\ Q & W_v^{-1} W_v^{-1} & 0 \\ Y^T & 0 & W_u^{-1} W_u^{-1} \end{bmatrix} > 0. \end{aligned}$$

The required state-feedback controller, $u = Kv$, is given by $K = YQ^{-1}$. Note that the initial state ellipsoid \mathcal{V}_0 can be obtained by transforming an ellipsoid in the original physical variables, and the weighting matrices are directly associated with physically quantifiable objectives.

The state, v , must be estimated from the relative position measurements, r . Our formulation guarantees the observability of v and we can use a completely analogous dual LMI problem to design an estimator gain matrix, L . For brevity we omit the formulation details.

5.2 - Reference tracking controller design

We now construct a reference tracking controller from the above estimator/state-feedback design of L and K , that exploits the redundancy in the relative position reference command.

We begin by using a singular value decomposition (SVD) to determine a (non-unique) matrix M satisfying $M^T r = 0$. The SVD will give a representation for C_v of the form,

$$C_v = [U_v \ U_z] \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} V^T,$$

and $M = U_z$ is one suitable choice. Given a commanded relative position, r_{cmd} , we wish to find a matrix, N_r , that gives a desired stationary state, $v_\infty = N_r r_{\text{cmd}}$, such that the system holds the commanded relative position vector (i.e. $C_v v_\infty = r_{\text{cmd}}$). We exploit the fact that r_{cmd} must specify a valid formation, i.e. $M^T r_{\text{cmd}} = 0$.

These requirements can be shown to be equivalent to the conditions, $A_v N_r = 0$ and $C_v N_r = (I - MM^T)$. Any N_r satisfying the equation, $[A_v^T \ C_v^T] N_r = [0 \ I - MM^T]$ meets these requirements. The complete relative position reference tracking controller is now given by the state-space representation,

$$\begin{aligned} \dot{w} &= [A_v + B_v K + L C_v] w + [-B_v K N_r \ -L] \begin{bmatrix} r_{\text{cmd}} \\ r \end{bmatrix} \\ u &= K w + [-K N_r \ 0] \begin{bmatrix} r_{\text{cmd}} \\ r \end{bmatrix}, \end{aligned}$$

where w is the controller state.

5.3 - Exploiting Input Redundancies

The linear model of an individual spacecraft's dynamics is essentially a double integrator. Force actuators—typically thrusters—are used for the control inputs, and these may have additional dynamics associated with them. If each spacecraft has zero order or identical first order actuator dynamics, then the input control space contains an additional degree of freedom. Note that if the actuators are reasonably similar servo loops can be used to give each spacecraft

equivalent actuation dynamics. We now demonstrate how this additional degree of freedom can be exploited to achieve other formation objectives.

Under the above assumptions, B_v has reduced column rank. The physical interpretation is that we need only control $N - 1$ of the spacecraft in order to control the relative positions of the formation. This means that there is a vector, B_\perp , satisfying $B_v B_\perp = 0$. An SVD can be used to calculate this vector, and we can define a projection, $(I - B_\perp B_\perp^T)$, such that,

$$B_v(I - B_\perp B_\perp^T)u = B_v u \quad \text{and} \quad B_z(I - B_\perp B_\perp^T)u = 0.$$

Note that the projected control input, $(I - B_\perp B_\perp^T)u$, drives the observable state, v , in the intended manner, but does not directly drive the unobservable state, z , which contains the dynamics of the centroid of the formation. If $A_{zv} \neq 0$ these may still be driven indirectly through the state v . Moreover, $(A_z, B_z B_\perp)$ is controllable and any control signal of the form $u = B_\perp \nu$ directly drives the z part of the state. We can therefore calculate control actuation signals of the form,

$$\hat{u} = (I - B_\perp B_\perp^T)u + B_\perp \nu,$$

which allow us to control the z and v components of the state independently. The input u controls the formation in the manner given in the previous sections and ν can be considered as a control variable for the formation centroid (and other common unobservable states). We now give two relevant uses for this control.

5.4 - Minimizing formation fuel consumption

The control variable ν can be chosen to minimize the total formation fuel use. At each time instant, given the formation actuation command u , we calculate ν as the solution to the following linear program.

$$\min_{\nu} \|(I - B_\perp B_\perp^T)u + B_\perp \nu\|_1, \quad \text{where} \quad \|\hat{u}\|_1 := \sum_{i=1}^N |\hat{u}_i|.$$

If actuator servo loops have been applied on each spacecraft then the u_i represent commanded thrusts and these are only approximately equivalent to the fuel used on each spacecraft. Note that this approach minimizes the total formation fuel consumption for a given controlled maneuver. It is not necessarily a solution to the problem of finding the minimum fuel maneuver between specified formation configurations.

5.5 - Control of the formation centroid

We now consider the problem of using the variable ν as a means of controlling the (unobservable) formation centroid. The dynamics of the unobservable state can be expressed as,

$$\dot{z} = A_z z + A_{zv} v + B_z B_\perp \nu.$$

By construction, $(A_z, B_z B_\perp)$ is controllable. We again take the approach of separating this control problem into an estimator and state-feedback design. The lack of observability of z means that the estimator is now open-loop and given by the z dynamics above.

Because v is a known—or estimated—quantity in the controller we pose the state feedback problem using knowledge of v . The controller then takes the form $\nu = -K_z \hat{z} - K_{zv} \hat{v}$, where \hat{z} and \hat{v} are the estimates of z and v respectively.

To begin we augment the z dynamics with a fictitious noise n as follows.

$$\dot{z} = A_z z + A_{zv} v + B_z B_\perp \nu + n.$$

Given that z is available only as an open-loop marginally stable estimate this is physically reasonable. It will also provide a means of tuning the control gains to accord more or less weight to the estimate of z . The objective is now specified as finding the minimum γ such that

$$\|z\|_2^2 + \|W_\nu \nu\|_2^2 \leq \gamma^2 (\|x\|_2^2 + \|W_n n\|_2^2).$$

This minimizes the \mathcal{H}_∞ gain between the noise (n) and measured state (x) disturbing z , and the unobservable state (z) and the control effort (ν). The symmetric positive definite weights, W_ν and W_n allow the designer to trade between the relative importance of z and ν and of x and n respectively. This problem is solved by the following LMI optimization.

$$\min_{\gamma, Q, X, K_{zv}} \gamma \quad \text{subject to:} \quad \gamma > 0, \quad Q = Q^T > 0, \quad \text{and,}$$

$$\begin{bmatrix} (X^T B_\perp^T B_z^T + B_z B_\perp X - Q A_z^T - A_z Q) & (B_z B_\perp K_{zv} - A_{zv}) & -I & -X^T W_\nu^T & -Q \\ (K_{zv}^T B_\perp^T B_z^T - A_{zv}^T) & \gamma^2 I & 0 & -K_{zv}^T W_\nu^T & 0 \\ -I & 0 & \gamma^2 W_n^T W_n & 0 & 0 \\ -W_\nu X & -W_\nu K_{zv} & 0 & I & 0 \\ -Q & 0 & 0 & 0 & I \end{bmatrix} > 0.$$

The feedback gains are $K_z = XQ^{-1}$ and K_{zv} . The unobservable state component of the controller can now be integrated into the previous reference tracking controller expressed below in terms of the controller required to run on the i^{th} spacecraft.

$$\begin{aligned} \begin{bmatrix} \hat{z} \\ \hat{v} \end{bmatrix} &= \begin{bmatrix} A_z - B_z B_\perp K_z & A_{zv} + B_z K - B_z B_\perp (B_\perp^T K - K_{zv}) \\ 0 & A_v + LC_v + B_v K \end{bmatrix} \begin{bmatrix} \hat{z} \\ \hat{v} \end{bmatrix} \\ &+ \begin{bmatrix} -B_z(I - B_\perp B_\perp^T)KN_r & 0 \\ -B_v KN_r & -L \end{bmatrix} \begin{bmatrix} H_0 & 0 \\ 0 & H_j \end{bmatrix} \begin{bmatrix} r_{\text{cmd}} \\ r \end{bmatrix}, \\ u_i &= E_i \begin{bmatrix} -B_\perp K_z & (I - B_\perp B_\perp^T)K - B_\perp K_{zv} \end{bmatrix} \begin{bmatrix} \hat{z} \\ \hat{v} \end{bmatrix} \\ &+ E_i \begin{bmatrix} -(I - B_\perp B_\perp^T)KN_r & 0 \end{bmatrix} \begin{bmatrix} H_0 & 0 \\ 0 & H_j \end{bmatrix} \begin{bmatrix} r_{\text{cmd}} \\ r \end{bmatrix}, \end{aligned}$$

The H_j matrices define the input switching of the relative position measurements. These can be precalculated, $H_0 = I$, $H_1 = X_1 M_1$, \dots , $H_j = I - X_j M_j^T$, and applied to switch between locally measured or communicated relative position measurements. The switching between the H_j measurement matrices can occur independently and asynchronously between the various spacecraft controllers.

This controller implements both control objectives (precise control of relative positions via state feedback on \hat{v} , and open-loop control of the formation centroid via feedback on both \hat{z} and \hat{v}) in a manner which ensures that the objectives do not interact. This approach could equally well be used to implement lower bandwidth and/or lower resolution feedback control of the formation centroid if a lower precision measurement of absolute position was available.

6 - CONCLUSIONS

Deep space formations can be defined in terms of the relative positions of all of the spacecraft, and in this paper we outline a method for using such a specification in the design of formation-wide control algorithms. The state-space decoupling approach given here allows for the design of controllers, using only relative position information, that achieve a formation-wide tracking objective. The controllers can be implemented in a decentralized manner, and each spacecraft can switch between various precalculated local measurement or communicated variable options for control.

It is also possible to exploit an additional degree of freedom in the formation to allow for other objectives. Examples include minimizing formation fuel use, or maintaining the inertial position of the formation centroid. These objectives can be achieved in a manner which does not interfere with the precise formation tracking control objectives.

7 - ACKNOWLEDGMENTS

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